Sovereign default risk and commitment for fiscal adjustment

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A B S T R A C T

This paper studies fiscal policy in a model of sovereign debt and default. A time inconsistency problem arises: since the price of past debt cannot be affected by current fiscal policy and governments cannot credibly commit to a certain path of tax rates, debtor countries choose suboptimally low fiscal adjustments. An international organization, capable of designing a contract that coaxes debtors into a tougher fiscal stance via the provision of cheap senior lending in times of crisis, can work as a commitment device and improve social welfare.

1. Introduction

The main role attributed to international institutions such as the IMF during a debt crisis is the provision of liquidity to countries undergoing temporary financial stress due to coordination problems. Arguably, official lending can in this case ward off speculative attacks by catalyzing private lenders and avoiding coordination failures. However, IMF and EU interventions following the recent European debt crisis do not seem to fit into the typical liquidity provision story. Greece, for one, was not facing temporary financing problems in 2011: with its debt to GDP ratio close to 165% and a nominal deficit of 10% of GDP, it was deemed insolvent by markets. And the logic of catalytic lending does not fit well in this case: private lenders were rushing for the exit at the very same time official financing was flowing in. This suggests a role for lending by international organizations that goes beyond the traditional liquidity interpretation.

In order to investigate this issue, this paper incorporates fiscal policy decisions in a sovereign default model along the lines of Eaton and Gersovitz (1981) and Arellano (2008). Those models assume away liquidity problems and are suitable to the study of incentives for sovereign debt repayment. However, most of this literature does not separate “country debt” from “government debt”, which renders them}

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2 Back in 2010–11, macroeconomists were asserting that Europe was undergoing a solvency crisis, not a liquidity problem. Feldstein (2010) put it simply: Greece “is insolvent and cannot service its existing debt”. Paul Krugman was quoted stating that “it’s basically inconceivable that there won’t be some significant losses on present value for bondholders of Greek, Portuguese and Irish debt” (Bloomberg, 2011) and was echoed by Otmar Issing. Earlier in 2011, Nouriel Roubini said: “I am afraid that Greece, more likely than not, isn’t just illiquid, but insolvent” (Roubiniblog, 2011).

3 In December 2010, Greek debt in private hands amounted to roughly 285 billion euros, but in December 2012 this had been reduced to 105 billion. During the same period, official lending increased from less than 30 billion to nearly 185 billion – mainly from the European Stability Mechanism (ESM). Official lending did not catalyze private lending, it substituted for it.

4 There are now many papers building on this framework. Recent references include Aguiar and Gopinath (2006), Alfaro and Kanczuk (2005), Cuadra and Sapriza (2008), Guimaraes (2011), Mendoza and Yue (2012) and Yue (2010).
inappropriate to study fiscal adjustment issues.\footnote{One important exception is Cuadra et al. (2010). Their model also analyses fiscal policy in a model of sovereign debt and default, but their objective is different. They develop a quantitative model that accounts for pro cyclicality in fiscal policy in emerging markets. Andreasen et al. (2013) also separate “country debt” and “government debt”, but they focus on political economy issues. There are also papers studying how the possibility of default (and the actual default) affect fiscal policy, but differently from here, the results from those papers rely on distortionary taxation (see, e.g., Pouzo and Presno, 2014).} We assume that sovereign debt is paid out of the government’s currently available resources and focus on fiscal policy decisions. One key implication of the model is that fiscal policy in debtor countries suffers from a time inconsistency problem. In consequence, senior official lending coupled with fiscal conditionality can work as a commitment device for fiscal adjustment that benefits the borrower. In our view, the way events unfolded in the recent European debt crisis sits well with this alternative interpretation.

In the model, a tighter fiscal policy reduces the amount of borrowing necessary to repay maturing debt. Lower borrowing needs make it less likely that the debtor country will default in the future, which in turn leads to cheaper borrowing in the present. The lower odds of default associated with higher taxation lead to higher expected payments to creditors and lower expected output losses for debtors. Creditors, for their part, are assumed to be competitive and always break-even in expected terms, so in equilibrium, the benefits from higher tax rates accrue to the debtor.

The problem, however, is that the time-consistent fiscal policy is suboptimal. Whenever the debtor government chooses fiscal policy, there is a stock of outstanding debt that was issued in the past at a given price. High tax rates reduce the incidence of default and thus positively influence debt prices, but since past debt is sunk that bears no benefit to the government’s budget — even though debt prices react in secondary markets. Therefore, compared with the solution under commitment, there is too little fiscal adjustment and too many default episodes in equilibrium. The excessive default incidence is priced in by creditors, so the debtor would like to pledge higher taxes in the future; only it cannot credibly do so since its incentives for fiscal adjustment once debt has been sold become weaker.

In theory one could devise private contracts to address this time inconsistency problem. For example, the debtor country could issue bonds stipulating transfers from the bond holder to the debtor in bad states of nature, conditional on tough fiscal policies having been implemented. This would in principle provide the necessary incentives for fiscal adjustment. Trouble is, this contract wouldn’t be workable in practice. The bond holder and debtor would have to know and agree that a certain state of nature had materialized, and the creditor would have to determine if the proper level of fiscal adjustment was accordingly implemented. The fundamental problem is that transaction costs and free-riding problems would prevent a market-based solution to the time inconsistency issue.

However, an international lender of last resort with seniority could implement a similar contract. An institution like the IMF faces no free-riding problem and would arguably have the proper incentives to evaluate the state of the indebted economy and the implementation of fiscal policy decisions. It could then lend resources to debtor countries in distress at subsidized rates upon observing tougher fiscal policies in place — an arrangement that resembles the conditionalities imposed by the IMF on debtors. The financial aid in times of crisis would provide debtors with the appropriate ex-post incentives for fiscal adjustment. Owing to the IMF’s seniority status, private creditors become junior lenders when the institution gets involved and hence are the ones who end up footing the bill of IMF’s subsidized lending. Hence, this arrangement mimics a transfer from creditors to the debtor conditional on tough fiscal policies.

IMF loans are usually conditional on countries meeting some fiscal targets. At first blush, the stringent fiscal packages requested by the IMF may look like undue international meddling in sovereign countries’ fiscal choices. However, in the model, ex-post incentives to tighten fiscal policy enhance total welfare ex-ante by diminishing the probability of costly defaults, and since that is reflected in bond prices, the welfare gains go to the borrower itself.

The model implies that following a shock that renders debt default a concrete possibility, fiscal policy would be too lax due to the time inconsistency issue, which would lead to excessively high default risk and interest rates. The model also highlights the role that could be played by a third party able to commit to lend money at subsidized rates in exchange for tougher fiscal policy in the debtor country. A deal along those lines would lead to a lower jump in market interest rates after the shock and, importantly, would improve the debtor’s welfare.

In our view, this model-based narrative closely resembles the unfolding of events in the case of the recent European crisis. The aftermath of the Lehman Brothers event in 2008 led to lower growth perspectives and ability to repay debt, turning sovereign default into a real possibility. In the following quarters, peripheral European countries could still borrow at relatively low interest rates: markets acknowledged an implicit arrangement between borrowers and official institutions, so they were expecting fiscal tightenings and, possibly, transfers to debtors. Expectations were indeed fulfilled: painful fiscal adjustments were implemented by this set of European nations after 2009. Finally, after 2011, when debt repayment faltered even with the extra fiscal effort, financial support flowed in from the European Community, the IMF and the European Central Bank in a variety of ways. At least qualitatively, official intervention in the European case matches the normative prescriptions from the model.

While the baseline model is simple, the results extend to more realistic settings. In particular, we show they are robust to the inclusion of a negative relation between output and tight fiscal policy and still hold when fiscal policy rigidities are included in the model. Additionally, we show that short-term debt attenuates the commitment problem, thus providing another rationale as to why emerging economies borrow short term.

The paper is organized as follows. Section 2 describes the model and Section 3 derives the results on time inconsistency. Section 4 shows how an international organization can help to overcome the problem. Section 5 discusses IMF lending and some developments in the Euro zone debt crisis under the light of the model and Section 6 presents the extensions. Conclusions are in Section 7.

2. The model

2.1. Environment

Consider a stochastic endowment economy governed by a benevolent sovereign able to access international capital markets and levy domestic taxes. The government maximizes its citizens’ utility, assigns no weight to foreign creditors’ welfare and cannot commit to repay its maturing debt. Time is discrete. The representative consumer in the debtor country has utility:

$$U = \sum_{t=0}^{\infty} \beta^{-t} (u(c_t) + g_t)$$

where $\beta$ is the time-discount factor, $c_t$ is consumption in period $t$, $u(.)$ is a strictly increasing and strictly concave function, and $g_t$ is government spending. Hereafter, time subscripts are omitted to simplify the notation. Both $c$ and $g$ have to be non-negative.

\footnote{This tacit agreement and its impact on market prices were noted by the popular press. A newspaper article in November 2009 stated that “the implicit guarantee from Europe’s biggest economy convinced bond markets that there was no need to fear a sovereign default” and added that “any bailout would probably come with draconian conditions” (New York Times, 2009). Along the same lines, Reuters (2009) commented on “markets’ calm over soaring debt in Euro zone members” and pointed to an implicit guarantee by the European Union.}
In each period, the debtor country’s representative agent receives a stochastic endowment \( y \) drawn from a probability density function \( f \), which is continuous with full support in the \( [y_t, y_t]\) interval, with \( y_t > y_t > 0 \). For simplicity, we assume that the variables \( y \) are independently distributed over time. It is also assumed that \( u'(0) = \infty \) and \( u'(y_t) < 1 \). The former means that a country will never allocate all of its resources to the provision of public goods and the latter ensures that in a country with no debt, the optimal \( g \) is always greater than zero.

Upon observing the realization of the stochastic endowment \( y \), the government chooses between repaying maturing debt and defaulting. After debt and default decisions have been made, at the last stage of any given period, the tax rate \( \tau^* \) prevailing in the following period is chosen. It is assumed that taxes can’t be altered within a certain period, hence private consumption equals net income:

\[
\text{but is not binding otherwise.}
\]

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The government raises debt with the following maturity structure: every period a fraction \( \phi \) of the outstanding stock comes due and \( 1 - \phi \) goes on to add to the next period’s debt pile. This is equivalent to assuming that the government issues debt with varying maturities, being \( 1 - \phi \) the ratio between the amount of debt coming due in \( t + 1 \) and the amount of debt coming due in \( t \).

As usual in this literature, default leads to output losses. Output net of default costs is denoted by \( \bar{y} \). If there is no default \( \bar{y} = y \); if the country chooses to default in period \( t \), output falls to \( \bar{y} = (1 - \gamma)y \) from \( t + 1 \) on, where \( y > 0 \). Moreover, upon defaulting the debtor is banished from capital markets. In subsequent periods, the punishment ceases with an exogenous probability \( \phi \). When the debtor reenters capital markets, it does so with an outstanding debt equal to \( v0 \), where \( v \) is a positive constant. \( \gamma \)

We assume \( 0 < \gamma^* < \phi < 1 \).

If the government starts period \( t \) with some inherited debt \( d, \phi d \) is the amount that has to be repaid in \( t \). If it opts to issue new debt \( b \) at an endogenous price \( q \), the total stock of debt one period ahead will be given by \( d' = (1 - \phi)d + b \). Government spending is thus given by:

\[
\begin{align*}
  g &= \tau y - \phi d + qb \quad \text{if the debtor has access to capital markets} \\
  g &= \tau y \quad \text{otherwise}
\end{align*}
\]

subject to \( g \geq 0 \). We will loosely refer to the condition \( \tau y - \phi d + qb \geq 0 \) as the “government budget constraint”. As usual, there is a maximum level of debt that prevents the country from running a Ponzi scheme but is not binding otherwise.

The private sector has no access to external capital markets and hence private consumption equals net income:

\[
\begin{align*}
  c &= \bar{y}(1 - \tau). \\
  \text{Creditors are risk neutral and } 1 + r^* \text{ is the gross risk-free interest rate. There is a perfect competition among foreign lenders so creditors always break even in terms of expected returns. We assume } \beta(1 + r^*) < 1, \text{ so that the country has incentives to borrow at the point of zero probability of default. Define } \pi_t, \text{ the probability debt } \phi d \text{ is repaid in the following period. Also define } q(d^*) \text{ as the price of this debt by the time it is issued, and } q(d^*) \text{ its price prevailing in the subsequent period if default did not occur. In case of no default, the value of 1 unit of debt in the following period is }
\end{align*}
\]

\[
\begin{align*}
  \psi + (1 - \phi)E[q(d^*)] \frac{\phi + (1 - \phi)E[q(d^*)]}{(1 + r^*)}.
\end{align*}
\]

If default does occur (which happens with probability \( 1 - \pi_t \)), the debtor is banned from capital markets for one period and then regains access in the following periods with a per period probability equal to \( \pi_t \).

The value of 1 unit of debt at the second period if the country had defaulted in the first is equal to:

\[
\begin{align*}
  v q(\nu d') \frac{\zeta}{(1 + r^*)} \left( \frac{1 - \zeta}{(1 + r^*)} + \frac{(1 - \zeta)^2}{(1 + r^*)^2} + \frac{(1 - \zeta)^3}{(1 + r^*)^3} + \frac{(1 - \zeta)^4}{(1 + r^*)^4} \right).
\end{align*}
\]

Putting both parts together:

\[
q(d') = \pi_t \left( \frac{\phi + (1 - \phi)E[q(d^*)]}{(1 + r^*)} \right) + (1 - \pi_t) v q(\nu d') \frac{\zeta}{(1 + r^*)} (\zeta + r^*). \quad (1)
\]

Note that this expression assumes that there are no varying degrees of seniority, so the debtor owes \( v \) for every previously contracted unit of debt when it regains access to financial markets. Seniority would affect the second term, the value of the bond in case of default.

The sequence of events in a given discrete time period is the following:

1. \( y \) is revealed;
2. \( \nu \) decisions about defaulting on maturing debt \( \phi d \) and floating new debt obligations \( b \) are made;
3. the tax rate prevailing next period \( \tau^* \) is chosen.

Debt and default decisions are made after \( y \) is observed but before \( \tau^* \) has been chosen, reflecting the idea that the country cannot commit to a certain level of taxes when it issues debt.

The value function associated with repaying debt is:

\[
V_p(\tau, d, y) = \max_b \left\{ u\left( (1 - \tau)y + \tau y - \phi d + qb + \beta V(\tau', (1 - \phi)d + b, y') \right) \right\} \quad (2)
\]

where \( \tau' \) maximizes

\[
u((1 - \tau)y + \tau y - \phi d + qb + \beta V(\tau', d', y'))
\]

taking \( d' = (1 - \phi)d + b \) as given. \( V \) is the maximum of two value functions:

\[
V(\tau', d', y') = \max \left\{ V_p(\tau', d', y'), V_0(\tau', d', y') \right\}.
\]

If the government opts to default, the value function is:

\[
V_0(\tau, d, y) = \max_{\tau'} \left\{ u\left( (1 - \tau)y + \tau y + \beta V_0(\tau', d, y') \right) \right\} \quad (4)
\]

where

\[
V_0(\tau', d, y') = \max \left\{ u\left( (1 - \tau')y' + \tau y' + \beta \left[ \xi V_0(\tau', d, y') + \xi V(\tau', d, y') \right] \right) \right\}
\]

and \( y' = (1 - \gamma)y \). The default punishment kicks in one period after the default decision. As a tie-breaking convention, we will suppose that the country repays debt if indifferent.

2.2. Discussion of assumptions

An important assumption of the model is that debt and default decisions are made before the tax rate is chosen. That corresponds to a world where a sizable part of debt has already been contracted when fiscal policy decisions that affect debt repayment are made. Moreover, debt contracts cannot be made contingent neither on tax policy nor on output, as in most instances in the real world.

We also assume that the tax rate \( \tau \) cannot be modified after \( y \) is revealed. This captures a well-known feature of tax regimes: they generally require some time to be altered. While real world governments do reset tax rates, significant lags often exist between a perceived need to collect more revenues and cash flowing into public coffers. Immediately

As documented by Sturzenegger and Zettelmeyer [2008], creditors usually recover a sizable part of their lending following default episodes.
changing taxation on private wealth would be akin to sheer expropriation. We incorporate this widely accepted feature as an assumption in our model. The important implication of this assumption is that fluctuations in y affect tax revenues available for debt payment, which is consistent with the data.

In many quantitative papers in the sovereign debt literature, default cost depends positively on y, the current realization of output. This feature nearly automatically generates default in bad times. By assuming (i) that output losses due to default occur only once and (ii) that y is unrelated to y, we are shutting down this channel here. Similarly, a utility function concave in g could induce default in bad times due to higher marginal benefits of public expenditures when these are low (because a low y means low tax revenues for a given tax rate τ). Linearity in g thus means that this channel too is not at work in our model. Together these two assumptions make it easier to isolate the default channel stressed in this paper, the “government budget constraint channel”, and they also allow us to obtain analytical results.

The assumption of β(1 + r′) < 1 gives a reason for the domestic country to borrow. It is a usual assumption in the literature and can be thought as a reduced form for other motivations for sovereign debt (say investment in public infrastructure). Linearity in g implies that the government would like to have all government spending as early as possible (this result is presented in Appendix A.1).

There is no consensus in the literature about the underlying sources of default costs. The usually mentioned channels include: losses from declining trade; increases in international borrowing costs or exclusion from financial markets; other costs related to reputational loss; unplanned redistribution of income; and liquidity problems that lead to a reduction in domestic investment. Regardless of the sources of default penalties, what is important for our purposes is that not repaying in full entails costs to the debtor, and these costs are deadweight losses. In order to have a role for seniority, a few features usually absent from dynamic models of sovereign debt were added. We assume that part of the debt is recovered, which makes the problem a bit more convoluted than with the usual assumption of no repayment in case of default. Without this assumption, being a senior creditor makes no difference: either all creditors are fully repaid or all creditors receive nothing. Moreover, the simpler one-period debt assumption needed to be shed. When debt contracted in the past periods comes due smoothly in future periods, IMF seniority affects the price of previously issued debt. Our maturity structure allows for all these effects of seniority but keeps the problem dynamically recursive.14

### 2.3. Benchmark case: uncontingent taxes and no debt

Here we describe a benchmark case in which the country has no access to capital markets, so there are no default concerns. A benevolent government chooses τ′ in order to maximize Eq. (3) with b = d = d′ = 0. The tax rate τ′ only affects the term EV(τ′, 0, y′) and since there is never default that can be written as:

\[
\int_{\gamma_L}^{\gamma_H} \left\{ u((1-\tau')y') + r'y' + \beta EV(\tau', 0, y') \right\} f(y')dy'.
\]

The first order condition of this simple problem is:

\[
\int_{\gamma_L}^{\gamma_H} \left\{ u'(1-\tau')u'(1-\tau)y' - 1 \right\} f(y')dy = 0.
\] (5)

Essentially, the expression above equates the expected marginal benefit of an additional unit of the public good to its marginal cost in terms of reduced private consumption. Under our assumptions the benchmark τ is larger than 0 and smaller than 1. If τ = 1, c = 0, so the marginal utility of consumption goes to ∞. On the other hand if τ = 0, c = y ≥ y₉, u'(c) ≤ u'(y₉) < 1, and hence the integral would be strictly positive.

### 3. Equilibrium

#### 3.1. Choice of debt and default

Default decisions are made following the realization of output y. If the country chooses to repay its debt, it also chooses debt (or assets) to be carried to the next period. Given inherited debt d, total liabilities carried over to the next period are d′ = (1 − φ)d + b, with qb being the amount of resources currently raised.

We start by showing that whether or not the government budget constraint binds are crucial for our results.

**Proposition 1.** When the constraint g ≥ 0 is slack, the derivatives of the value functions with respect to y are

\[
\frac{dV_p}{dy} - \frac{dV_d}{dy} = u'(y)(1-\tau)(1-\tau) + \tau.
\]

Hence lifting the constraint g ≥ 0, there is never a default.

**Proof.** See Appendix A.3.

As long as g > 0, shocks to y affect Vₚ and Vₐ in exactly the same way: lower y diminishes private and public consumption to the same extent (since τ is predetermined). But this is so only as long as variations in y have no bearing on the choice of qb, which matters for Vₚ but not for Vₐ.

If the difference between Vₚ and Vₐ is not subject to shocks, then default is either a zero-probability event or a certain event. The latter is ruled out because it implies q = 0 and it is never optimal for the debtor to sell debt at zero price (the debtor would be better off by borrowing nothing and avoiding default punishment).

Since τ, y and d are predetermined, only qb can affect the constraint τy − ad + qb ≥ 0. Differentiating qb with respect to b yields

\[
\frac{\partial (qb)}{\partial b} = q(1-\epsilon) \quad \text{where} \quad \epsilon = -\frac{\partial q}{\partial b} \frac{b}{a}.
\]

**Lemma 9** (in Appendix A) implies that the probability of repayment is (weakly) decreasing in the level of debt. Hence extra debt Δb will tend to increase the inflow of resources by less than Δqb because the increase in b adversely affects the price of debt q. The size of this effect on the price of debt is given by the elasticity parameter ϵ.

The following proposition states when default can occur.

**Proposition 2.** Default occurs when tax revenues are low:

1. Default can only happen in bad times; for given τ and d, if there is default for some y ∈ [y₉, yₐ] then there is default if and only if y < yₗ for some yₗ ∈ [y₉, yₐ].

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14 Another consequence of our maturity structure is that new debt makes old debt less valuable as in Hatchondo et al. (2012), but this problem is not the focus of this paper.

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9 Empirical evidence includes Rose (2005) and Martinez and Sandleris (2011).

10 Exclusion from international capital markets is a usual assumption in models following Eaton and Gersovitz (1981). Empirical evidence can be found in English (1996) and Fuentes and Saravia (2010).

11 Foruz (2007) argues that reputational concerns are important for default decisions.

12 See, e.g., Broner and Ventura (2011).

13 See Brunti (2011) for a model. The survey in Panizza et al. (2009) highlights the importance of these domestic costs.

14 The i.i.d. assumption is important here: if y and y′ are linked, output shocks would also affect current value functions through their impact on the expected value of future value functions.
2. Default is more likely when \( \tau \) is smaller: for given \( y \) and \( d \), if there is default for some \( \tau \in [0, 1] \) then there is a default if and only if \( \tau < \tilde{\tau} \) for some \( \tilde{\tau} \in [0, 1] \).

3. If the probability of default next period is positive, than it is strictly decreasing in \( \tau \).

**Proof.** See Appendix A4. ■

A country that repays maturing debt also chooses new debt, \( b \). Since default occurs for \( y < \bar{y} \), the value function in Eq. (2) becomes:

\[
V_p(\tau, d, y) = \max_b \left\{ \frac{u(y(1-\tau)) + \tau y - \phi d + qb}{\beta} \int \left[ \int_{\tau} V_p(\tau', \bar{y}) f(y') dy' + \int_{\bar{y}} V_d(\tau', (1-\phi)d + b, y') f(y') dy' \right] \right\}
\]

(6)

where \( \tau' \) is given by Eq. (3). The choice of \( b \) depends on whether the government budget constraint binds. If it does not bind, then \( \tau y - \phi d + qb > 0 \) and we obtain the following first order condition:

\[
q(1-\epsilon) + \beta \int f(y) \frac{\partial V_p(\tau', (1-\phi)d + b, y')}{\partial b} f(y) dy = 0
\]

(7)

where \( b_y \) is the level of borrowing chosen if the constraint \( g \geq 0 \) is not binding. In the case where the constraint binds,

\[
b_{\text{min}} = \frac{\phi d - \tau y}{q}.
\]

A lower \( \tau y \) implies a larger minimum level of borrowing \( b_{\text{min}} \) in case the country pays its maturing debt.

The intuition for the first statement of Proposition 2 is the following: in bad states of nature (low \( y \)) the government constraint binds with \( g = 0 \), which leads to a larger reduction in \( V_p \) in comparison to \( V_d \). The optimal unconstrained borrowing \( b_y \) is not feasible anymore when output turns out too low and the government chooses to repay. Additional resources need to be raised in capital markets to make up for the shortfall in revenues if the country decides to honor its debt. The constrained level of borrowing \( b_{\text{min}} > b_y \) will then be suboptimally large in case of repayment. Higher financing needs translate into a larger probability of default, which drives down the price of debt (or, analogously, drives up interest rates). That does not imply that the debtor will be transferring more money to creditors, because creditors always break-even in expected terms. From an accounting perspective, the increase in interest rates and the decrease in the probability of repayment cancel each other. However, from an economic perspective, the decrease in the probability of repaying increases the probability of facing output costs in the future. Hence an increase in financing needs effectively renders the option of repaying relatively more costly.

The intuition for the second statement is essentially the same. Tax revenues are given by \( \tau y \), so low realizations of \( y \) and low values of \( \tau \) play a similar role in the choice of debt and default. For the third statement, note that higher tax rates imply a lower threshold \( \tau \) for default (the set of values for \( y \) that leads to default is smaller), and since the distribution of \( y \) has full support, a larger \( \tau \) leads to a smaller probability of default.

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10 A change in \( b \) could also affect the value function through its effect on the choice of \( \tau' \). Indeed, the derivative of \( V_p \) with respect to \( b \) includes the term

\[
\frac{\partial EV(\tau', d', y')}{\partial b} \frac{\partial \tau'}{\partial b}
\]

but since \( \tau' \) will be optimally chosen, the first derivative term is zero.

3.2. The choice of fiscal policy

We focus on the case the debtor has access to capital markets, so there might be risk of default in the following period. The optimal tax rate \( \tau \) in equilibrium comes from maximizing Eq. (3) taking the level of debt for the following period \( d' \) as given (owing to the assumptions on timing). The first order condition with respect to \( \tau \) is simply:

\[
\frac{\partial EV(\tau', d', y')}{\partial \tau} = 0.
\]

(9)

4. The time inconsistency problem

We now show the choice of fiscal policy is time inconsistent. Consider the choices at \( t_0 \) of a government (i) in case it can commit to a tax rate \( \tau \) when issuing debt in period \( t_0 \); and (ii) in case it chooses a tax rate \( \tau' \) at \( t_0 \) after debt has been contracted, as in Section 3. We will show that when the default risk is relevant, \( \tau > \tau' \). The comparison takes as given the possibility of commitment in the future: the country can commit to choose a certain tax rate \( \tau_t + 1 \) when issuing debt \( b_t \) in periods \( t \in T_\infty \), where \( T_\infty \subset \{t_0, t_0 + 1, t_0 + 2, \ldots \} \), and cannot commit otherwise. For the sake of expositional simplicity, we consider in the main text the case where there will be no opportunity for commitment in the future, so \( T_\infty = \emptyset \). The general case is dealt with in Appendix A6.

Suppose the government can commit to some tax rate equal to \( \tau \) when it issues debt \( b \) at the current period. The value function in case of default is unchanged as in Eq. (4). The value function in case of repayment becomes

\[
V_p(\tau, d, y) = \max_{\tau} \left\{ \frac{u(y(1-\tau)) + \tau y - \phi d + qb + \beta EV(\tau, (1-\phi)d + b, y)}{\beta} \right\}
\]

(10)

where \( q \) is now a function of both \( d' \) and \( \tau' \). The expression for \( q \) is still given by Eq. (1) but now changes in \( q \) affect the price of debt through the probability of default and the price of debt in the following period (\( \tau_t \) and \( q(d' \)). Note that the continuation value function \( EV(\tau, (1-\phi)d + b, y) \) is the same as in Eq. (2) since there are no differences in the possibility of commitment in the future.

The next proposition establishes that \( \tau \) positively affects the price of debt.

**Proposition 3.** If the probability of default next period is positive and the debtor can commit to a tax rate \( \tau \) when issuing debt \( b > 0 \),

\[
\frac{\partial q}{\partial \tau} > 0.
\]

**Proof.** See Appendix A5. ■

An increase in \( \tau \) increases the probability of repayment for the reasons discussed in Proposition 2, and that is reflected in the price of debt \( q \). Higher taxation means less refinancing needs and thus smaller chances of defaulting in the future. This is fully priced into interest rates given the assumption of perfectly competitive capital markets.

We now show that if the debtor could commit to a certain tax rate \( \tau \), the chosen \( \tau \) would be larger than \( \tau' \) given by Eq. (9). In this case, \( \tau \) maximizes the expression for \( V_p(\tau, d, y) \) in Eq. (10), taking into account the effect of \( \tau \) on \( q \). The solution to the maximization problem for \( \tau \) depends on whether the constraint \( g \geq 0 \) binds. Suppose first it does not bind. Then \( \tau y - \phi d + qb > 0 \) and the first order condition with respect to \( \tau \) yields:

\[
\frac{\partial EV(\tau, d', y')}{\partial \tau} = 0.
\]
now
\[ \frac{\partial (q_b)}{\partial T} - \frac{\partial (q_b)}{\partial \tau} + \frac{\partial (q_b)}{\partial \tau} \frac{\partial b}{\partial T} = b \frac{\partial q}{\partial T} + q(1-\epsilon) \frac{\partial b}{\partial T} \]
so we get:
\[ b \frac{\partial q}{\partial T} + \beta \frac{\partial b}{\partial \tau} \left( q(1-\epsilon) + \beta \int_y \frac{\partial V_p(\tau, (1-\phi)d + b, y)}{\partial b} f(y') dy' \right) \]
\[ + \beta \frac{\partial EV(\tau, d', y')}{\partial T} = 0. \]

The optimal level of borrowing is still given by the expression in Eq. (7). Substituting that in the above expression yields:
\[ b \frac{\partial q}{\partial T} + \beta \frac{\partial EV(\tau, d', y')}{\partial \tau} = 0. \]
(11)

The choice of \( \tau \) with no commitment is given by Eq. (9). The expression in Eq. (11) shows that when the government can commit to a tax rate before issuing debt, \( \tau \) has an extra positive effect on the value function, since it positively affects the debt price \( q \). This term is particularly important when \( b \) is large.

Now suppose that the constraint \( g \geq 0 \) binds so that \( rT - \phi d + qb = 0 \). Taking the derivative of \( b \) from Eq. (8) with respect to \( \tau \) yields
\[ \frac{\partial b}{\partial \tau} = \frac{(-\phi d - rT) \frac{\partial q}{\partial \tau}}{\partial q^2} \]

hence the first order condition with respect to \( \tau \) implies
\[ \beta \left( -\frac{(\phi d - rT) \frac{\partial q}{\partial \tau}}{\partial q^2} \right) \int_y \frac{\partial V_p(\tau, (1-\phi)d + b, y)}{\partial b} f(y') dy' + \frac{\partial EV(\tau, d', y')}{\partial \tau} = 0. \]
(12)

The first term of the LHS in Eq. (12) is positive (the derivative inside the integral is negative and all other terms are positive with a negative sign). Again, compared with the expression in Eq. (9), \( \tau \) has an extra positive effect on the value function in case of commitment owing to its effect on the debt price. In case the constraint \( g \geq 0 \) binds, that reduces the need for borrowing.

The above formulation of the problem assumes no possibility of commitment in the future, but the result does not rely on that assumption. Proposition 4 shows the general result.

Proposition 4. Consider the choices at \( t_0 \) of a government (i) in case it can commit to a tax rate \( \tau \) when issuing debt in period \( t_0 \); and (ii) in case it chooses a tax rate \( \tau \) at \( t_0 \) after debt has been contracted. For both cases, suppose the country can commit to choose a certain tax rate \( \tau_{t+1} \) when issuing debt \( b_t \) in periods \( t \in T_c \), where \( T_c \subset \{t_0 + 1, t_0 + 2, \ldots, \infty\} \), and cannot commit otherwise. If default risk is relevant and \( b_{t_0} > 0 \), then \( \tau > \tau^* \).

Proof. See Appendix A6.

Intuitively, if commitment is feasible, a pledge to implement a certain tax rate \( \tau \) will be fully incorporated in bond prices, which increases the incentives for additional fiscal adjustment. The inability to commit to a future level of taxes coupled with the timing structure (the tax rate is chosen when debt has already been contracted) is what renders the equilibrium tax rate suboptimal.

It is the debtor itself that benefits from higher taxes since these mean less frequent defaults, which are costly events. Debtors do internalize the fact that higher taxes lead to less frequent defaults when deciding how much to tax the private sector endowment. The problem, however, is that as a consequence of time inconsistency, this internalization is incomplete. A pledge of high future tax rates is judged to be not credible because ex-post, with the price of debt already set, debtors’ incentives are altered: they do not reap additional benefits in terms of lower interest rates by sticking to any promised higher tax rate. If debtors could somehow commit to higher taxes, they would fully benefit from the effects of these on interest rates. The time inconsistency problem is then unfortunate to them, leading to smaller welfare via suboptimally frequent default events.17

5. A commitment device for fiscal adjustment

From an ex-post point of view, the choice of fiscal policy entails an externality: the debtor internalizes the cost it will face from defaulting but not the loss incurred by the creditors. Hence there is room for a deal that improves welfare in the economy, in the spirit of the Coase theorem. For instance, lenders could transfer resources to the debtor if a more stringent fiscal stance were chosen (since that would increase the odds of repayment). However, from an ex-ante point of view, lenders always break even in the model: any transfer as well as any change in the probability of default would be reflected in the price of debt. Hence the externality is actually a time inconsistency problem, it is the debtor itself who will benefit from committing to a choice of taxes that reduce the odds of default.

We now show one way for the debtor to commit to a more stringent fiscal stance, discuss the problems of a private contract along those lines and the role of an international organization in implementing the deal.

5.1. A private contract with lenders

Let the equilibrium (time-consistent) tax rate be \( \tau^* \). Now consider a debt contract such that the debtor country receives a transfer \( T \) from the lender contingent on choosing \( \tau = \tau^* + \Delta \tau \). The price of debt is now given by:
\[ q(\tau') = \pi_1 \left[ \phi + (1-\phi)E_q(d') \right] + (1-\pi_1) \frac{\nu q(1d')}{(1+r')} \frac{\zeta}{(1+r')/(1+r)} - \frac{T}{1+r}. \]
(13)

This expression seems very similar to Eq. (1), the only difference seems to be that now lenders take into account that they will pay a transfer \( T \) to the debtor country in the following period. However, as long as the contract affects the choice of fiscal policy, it will affect the probability of repayment \( \pi_1 \) and, consequently, the price of debt.

The following proposition shows that a contract along these lines improves welfare.

Proposition 5. A private contract with lenders:

1. There is some \( T > 0 \) and \( \Delta \tau > 0 \) such that a contingent debt contract that entitles the debtor to receive a transfer \( T \) contingent on choosing \( \tau^* + \Delta \tau \) improves welfare in the debtor country and leaves the lenders unaffected.

2. A contract with transfers only in states where the constraint \( g \geq 0 \) is binding leads to a larger welfare improvement than a contract with an uncontingent transfer of the same average magnitude.

Proof. See Appendix A7.

The first statement of the proposition shows one way that the borrower can buy commitment: this debt contract also entails a loan of \( T/(1+r) \) from the borrower to the lender at the risk-free rate.17

17 Hatchondo et al. (2012) study a different time-consistency problem: since new debt reduces the value of previously issued debt, the government would like to commit to issue less debt. As in this paper, the government cannot commit to policies that would increase the value of debt issued today, but the nature of the problem studied in Hatchondo et al. (2012) is different and so are the policy prescriptions. They study the inclusion of covenants in debt contracts that limit debt issuance.
Crucially, the debtor will only be repaid if it chooses a tighter fiscal stance. That has a positive effect: it works as a commitment device for the debtor, leading to the choice of a fiscal stance that is closer to the commitment solution. It also has a negative effect: since the debtor receives \( T \) only in the following period, it might have to borrow more in the current period to roll over its debt (in case the \( g \geq 0 \) constraint is binding), and borrowing in this case is expensive. However, for a small \( \Delta \tau \), the transfer \( T \) is of second order of magnitude because the debtor is close to indifference between \( \tau^* \) and \( \tau^* + \Delta \tau \), so the negative effect is dominated by the positive effect. For larger values of \( \Delta \tau \), the negative effect might become important, so the commitment solution might not be implemented by this particular scheme.\(^1\) Nevertheless, this mechanism can always improve upon the time-consistent solution. Debt is always contracted at its actuarially fair price, so lenders always break even in expected terms.

In sum, the transfer from creditors to the debtor are offset by a negative effect on bond prices. The benefits from this contract stem from its effect on fiscal policy. An expected tighter fiscal stance leads to a lower probability of default and thus to higher bond prices, which benefits the debtor.

The intuition for the second statement is related to the intuition for larger effects of \( \gamma \) on the value function \( V_\gamma \) when the government budget constraint is binding, discussed in Proposition 2. If \( g > 0 \), a transfer of resources \( \Delta T \) to the debtor leads to an increase in \( g \) by \( \Delta T \) and the marginal utility of this higher public spending is one. In contrast, when the \( g > 0 \) constraint binds, the transfer \( \Delta T \) is used not to increase \( g \), but to reduce borrowing and relax the binding constraint. An extra gain arises because a reduction in \( b \) leads to a reduction in the odds of default and, consequently, an increase in the price of debt \( q \). The implication is that transferring resources to the debtor when they will be used to repay maturing debt provides more incentives to the debtor for each dollar it receives.

5.2. The IMF as a commitment device for fiscal adjustment

The private contract discussed above is problematic because of the transaction costs involved. Lenders must commit to the transfer contingent on a certain level of fiscal adjustment, which has to be chosen and then verified. In reality, that requires some degree of commitment and a large amount of effort. Individual creditors would have incentives to free ride on others’ monitoring efforts. Overcoming the time inconsistency problem discussed above through private contracting seems to be unfeasible. However, the problem could be solved by a big long-term player that is able to (i) choose and verify the optimal level of fiscal adjustment; (ii) commit to transfer resources to the debtor after observing the implementation of the chosen level of fiscal policy; and (iii) take resources from the lenders to fund this transfer.

We think of the IMF and the EU as institutions able to carry out this type of implicit contract. An institution like the IMF can send a mission to a country, determine what the fiscal stance \( \tau^* \) should be and verify the implementation of the contracted fiscal adjustment (the first point above).

Contingent on the implementation of the contracted \( \tau^* \), the IMF can commit to transfer resources to the debtor country (second point above) by lending money at below market rates. The second statement of Proposition 3 shows that transfers should be given in times the \( g \geq 0 \) constraint is binding. An institution like the IMF can commit to this contingent lending deal, providing large loans at low rates to a country that has chosen a tight fiscal stance, but only when it turns out that the country got an unlucky draw of \( y \) and needs support to roll over its debt.

The IMF attaches seniority clauses to its lending, implying that creditors that had previously lent to that country become juniors. In case of default, official and private lenders would eventually receive some payments because when the debtor regains access to markets, lenders are entitled to a proportion \( v \) of their previously contracted debt. Seniority implies that the IMF will then be repaid in full, and other lenders will be entitled to whatever is left after the IMF has been paid. Therefore, IMF seniority is equivalent to a transfer from lenders to the IMF (third point above). The transfer is not made when the IMF lends to the debtor, but afterwards, and only in case of default. Hence, IMF breaks even and its lending is virtually risk free.

In the language of the model, the price of debt paid by lenders is indeed given by Eq. (13). In the following period, the IMF lends money to the debtor country at risk-free rates in states that the constraint \( g \geq 0 \) is binding. That is effectively a transfer to the debtor country and its expected present value is \( T \). In case default happens, the lenders transfer some of their rights to the “scrap value” of debt to the IMF. Since the IMF breaks even, the expected present value of this transfer is \( T \). The share of payment going to the lenders is adjusted to make sure the IMF is fully paid.\(^1\)

In our view, this kind of agreement resembles some of the real world deals we observe, including IMF programmes and the recent deals struck in the Eurozone. We now turn to these.

6. Discussion

6.1. IMF adjustment programmes

The results in this paper point to a re-interpretation of IMF adjustment programmes, with 3 key elements: (i) lending at below market rates coupled with (ii) fiscal conditionalities and (iii) IMF seniority. These three ingredients combine to generate a commitment device for fiscal adjustment that benefits the debtor while the IMF breaks even.\(^2\)

The fundamental problem in the model is that the time-consistent choice of taxes is too low, the debtor does not internalize the effect of fiscal policy on the price of debt that has already been issued. The IMF can solve this problem by offering loans at below market rates conditional on a tougher fiscal policy. Hence, conditionality is an intrusion in the sovereign government that helps correct a time inconsistency problem that renders taxes too low and default too frequent.\(^2\) The correction raises bond prices and hence improves welfare in debtor countries. The IMF is often criticized for imposing excessively tough measures on debtor countries. Summarizing this view, Dreher (2009) states that “if the Fund and the government disagree about appropriate policies, it is the government that should have the final say, and not the Fund”. That is not true in the model because the debtor is time inconsistent.

---

\(^1\) This requires a large enough recovered value of debt \( v \) in order to cover all IMF’s lending. Hence the amount of incentives for fiscal adjustment that can be provided is also constrained by \( v \).

\(^2\) Bird (2007) provides a comprehensive survey on the debate about the role the IMF should play and how it ought to operate. The usual function attributed to the IMF is the provision of loans to countries facing liquidity problems, but the literature also points out that IMF programs can help domestic leaders to signal their types to financial markets in a setting of asymmetric information, and to overcome internal political barriers to reforms (see, e.g., Drazen (2002) and Vreeland (2003)).

\(^3\) There is a debate about IMF conditionalities. Dreher (2009) presents several other possible rationales for them: (i) a way to transfer resources to creditors in detriment of debtor’s welfare; (ii) a mechanism through which countries can “signal their type” to markets; (iii) a way for domestic leaders to implement politically difficult but beneficial policies that face the opposition of some well-organized groups inside the country; (iv) the perhaps only way benevolent international actors have to impose “good policies” on poorly informed domestic policymakers; and (v) as commitment devices for investing rather than consuming (Sachs (1989)) or for undertaking policies that increase the costs of default (Fafchamps (1996)).
Loans at below market rates coupled with fiscal conditionalities help the debtor to commit to a tough fiscal stance, but would be costly to the IMF. That is where the IMF’s status as a senior lender kicks in.22 IMF seniority plays the role of a transfer from creditors to the IMF, which is part of a deal linking the IMF, creditors and the debtor country. From the IMF point of view, seniority ensures it breaks even in expected terms. The transfer from creditors is reflected on bond prices, and competition among them ensures that lenders also break even in expected terms.

It is conceivable that the time inconsistency problem highlighted in the paper could be attenuated by a policy maker’s reputation for fiscal toughness. However, the process of building reputation can be painstakingly long for a highly indebted country in need of resources to roll over debt. By providing ex-post incentives for a tough fiscal policy, a deal with the IMF might make a fiscal adjustment plan credible, thus reducing incentives for sovereign default in the future and increasing bond prices ex ante.

As a concrete example, consider the case of Brazil in 2002. Back then Lula, a former union leader identified with left-wing policies, became president. With a net public debt above 60% of GDP, it was crucial for Lula’s administration to quickly convince financial markets that it would pursue an austere fiscal policy course. Expectations of a loose fiscal stance would have implied very low bond prices and, possibly, financial mayhem. The Brazilian government hastened to get a deal with the IMF, thereby promising a fiscal primary surplus of 4.25% of GDP in exchange for the IMF support if needed. This deal with the IMF did manage to convince markets: in a few months, Brazil’s EMU risk premium came tumbling down from more than 1000 basis-point to nearly half of this value.23

The analysis in this paper is restricted to fiscal policy. As Dreher (2009) and Bird (2007) point out, IMF conditionality has often included a wide set of policies, about which this paper has nothing to say. Moreover, there is also a debate about whether IMF adjustment programmes effectively affect policies (see Easterly (2005) and the discussion in Dreher (2009)), and again, we have nothing to add to that discussion.

6.2. The European sovereign debt crisis

It is difficult to reconcile the unfolding of events in Europe following the financial crisis of 2008 with a view that attributes to official lenders the role of providing funds to cover short-run liquidity needs. The model provides an alternative interpretation: official intervention helped debtor countries to commit to a tighter fiscal stance. As a result, they were able to borrow at relatively low rates when the financial crisis of 2008 erupted. Consistently with this story, the indebted European countries have undertaken fiscal adjustment measures since 2009, and have received subsidized lending since 2011.

Following the 2008/2009 financial crisis, the probability of sovereign defaults in peripheral Europe increased due to depressed growth, lower revenues and the need to shore up banks with public money.24 However, the actual interest rate increase that ensued looks modest in retrospect. In 2009, interest paid on a 10-year debt from Greece, Portugal, Spain, Italy and Ireland were on average at 189, 68, 59, 75, 145 basis points above German’s 10-year bond, respectively. The relatively small rise in interest rates is consistent with the anticipation of higher tax rates to avert a costly default. Many countries (Ireland, Greece, Spain, Portugal, Italy) either received bail-out funds from multilateral lenders (IMF and the EU) or were de facto rescued by unconventional monetary policy operations implemented by the ECB. Since 2011, roughly 200 billion Euros were granted by the EU and the IMF to Greece alone. These loans can be seen as the promised carrot giving debtor countries the incentives to implement a tougher fiscal effort — and, indeed, they were only granted after an observed tightening in fiscal policy. Moreover, through the Long Term Refinancing Operation (LTRO), the ECB provided funds to the banking system for 3 years at 1% per year. This huge operation (amounting to 1 trillion Euros) constituted an indirect help for sovereigns committed to shoring-up their banking systems, and national banks were allegedly coerced into using these funds to buy government’s debt (the so-called Sarko trade). Further, in 2012, the ECB announced the Ongoing Monetary Transactions (OMT) programme aimed directly at financially helping governments. The OMT’s design entailed unlimited resources but stringent fiscal conditionalities.

In sum, the crisis resolution in Europe seemed to have followed a path consistent with an implicit contract through which multilateral lenders provide countries with subsidized money in return for a tough fiscal policy.25 According to the model, fiscal policy would indeed be inefficiently loose in the absence of any intervention.

7. Extensions

We now extend the model in order to consider (i) a negative relation between output and tight fiscal policy; (ii) fiscal policy rigidities; and (iii) short-term debt. For the sake of notational simplicity, we will assume that the government will not be able to commit to a tax rate when issuing debt at any point in the future. However, as discussed in Appendix A.6, the results can be generalized to consider future possibilities of commitment.

7.1. The austerity × productivity tradeoff

The announcement of fiscal consolidation plans in the European countries following the debt crisis of 2011 sparked a heated debate about the benefits of austerity. The concern was that a combination of smaller government spending and higher tax rates could contribute to a reduction in output, either because tight fiscal policy would adversely affect aggregate demand or because distortionary taxation and insufficient public investment would hinder productivity. This section shows that adverse effects of fiscal tightening on output do not affect the time inconsistency problem highlighted in this paper. The negative

22 The trade-off involved is studied by Saravia (2010): senior lending allows the IMF to provide more liquidity, but the possibility of large senior lending in the future might reduce ex-ante incentives for private lending.

23 As in the model, IMF conditional lending at below market rates would come at the expense of private lenders becoming junior. However, by providing incentives for tight fiscal policy, the deal with the IMF contributed to reduce interest rates paid on Brazilian debt, allowing the government to weather out a confidence crisis.

24 Default probabilities were low until Lehman Brothers, with 10-year Greek bonds paying around 5% a year close towards the end of 2008. By mid-2010, this figure had doubled.

25 Quoting Barry Eichengreen, “the crisis countries have, in fact, shown remarkable resolve in implementing painful cuts” (http://www.project-syndicate.org/commentary/eichengreen25-English).

26 Data from Eurostat.

27 That was summarized by the ECB’s chair, Mario Draghi: “What I believe our economic and monetary union needs is a new fiscal compact — a fundamental restatement of the fiscal rules together with the mutual fiscal commitments that Euro area governments have made. (…) Other elements might follow, but the sequencing matters”. http://www.ecb.int/press/key/date/2011/html/sp111201.en.html.
effects of tight fiscal policy on the economy reduce both the optimal time-consistent tax rate and the optimal tax rate under commitment, but the latter is still higher than the former.

This section extends the baseline model in order to incorporate this adverse outcome effect of fiscal contraction in a reduced form way. We assume the output is given by $y(h)$, where the function $h$ captures the negative effects of taxes on output loss. The function $h$ satisfies $h(0) = 1$, $h(1) = 0$, $h' < 0$ and $h'' < 0$. The assumption of $h'' < 0$ captures the idea of convex tax costs: larger tax rates lead to more distortions at the margins.

The basic logic underlying the time inconsistency problem in the baseline model remains intact because the optimal choice of tax rate internalizes its impact on output. The negative effect of tight fiscal policy on output leads to lower tax rates with and without commitment. As before, without commitment, the government remains unable to internalize the effects of $\tau$ on bond prices and hence the optimal tax under commitment $\bar{\tau}$ is greater than the time-consistent tax rate $\tau^*$. In conclusion, the claim that fiscal policy in Europe should take into account the adverse effects of taxation on output does not invalidate the point that in the absence of an arrangement designed to overcome the time inconsistency problem, fiscal policy will all the same remain too loose. However, this paper has nothing to say on the comparison between optimal tax rates and those observed in the real world. Critics of the European austerity plans argue that fiscal policy has been excessively contractionary. Some economists even claim that current tax rates are at the wrong side of the Laffer curve and have actually reduced defaults more likely.28 Policy blunders may occur and official lenders (together with borrowers) may enact self-defeating fiscal packages, but this paper has nothing to add to this discussion.

7.2. Short term debt and fiscal policy rigidity

This second extension deals with the possibility that taxes remain unchanged for a longer time span. The tax rate is chosen only in odd periods, and holds for two periods. There are two ways to interpret this extension: (i) a time period is as before, but now taxes are only changed every other period; and (ii) a time period now is half of what it was before, the frequency of changes is now larger than 1. As before, the marginal cost of taxation for government spending, but also by depressing output. Note that as $\tau \to \tau^*$, the derivative of $h(h(\tau^*))$ with respect to $\tau$ approaches 0, so the marginal cost of taxation goes to infinity, hence the government always chooses $\tau < \tau^*$.

This modification does not affect the qualitative results of the model, as summarized in the next proposition.

Proposition 6. In the model where output is given by $y(h(\tau))$:  
1. If the probability of default next period is positive, and the debtor can commit to a tax rate $\tau$ under commitment, the government will never choose $\tau > \tau^*$.
2. If the probability of default next period is positive, and the debtor can commit to a tax rate $\tau$ when issuing debt, then $\tau > \tau^*$.

Proof. See Appendix A.8.

The basic logic underlying the time inconsistency problem in the baseline model remains intact because the optimal choice of tax rate internalizes its impact on output. The negative effect of tight fiscal policy on output leads to lower tax rates with and without commitment. As before, without commitment, the government remains unable to internalize the effects of $\tau$ on bond prices and hence the optimal tax under commitment $\bar{\tau}$ is greater than the time-consistent tax rate $\tau^*$. In conclusion, the claim that fiscal policy in Europe should take into account the adverse effects of taxation on output does not invalidate the point that in the absence of an arrangement designed to overcome the time inconsistency problem, fiscal policy will all the same remain too loose. However, this paper has nothing to say on the comparison between optimal tax rates and those observed in the real world. Critics of the European austerity plans argue that fiscal policy has been excessively contractionary. Some economists even claim that current tax rates are at the wrong side of the Laffer curve and have actually reduced defaults more likely.28 Policy blunders may occur and official lenders (together with borrowers) may enact self-defeating fiscal packages, but this paper has nothing to add to this discussion.

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Let $V^1$ be the value functions in odd periods and $V^2$ the value function in even periods. The value function associated with repaying debt in odd periods is:

$$V^1_1(h(\tau), d, y) = \max_{\tau} \{ u(\tau h(\tau) y) + \tau h(\tau) y - \phi d + q b + \beta EV(\tau, (1-\phi) d + b, y') \}$$

where $\tau$ maximizes

$$u(\tau h(\tau) y) + \tau h(\tau) y - \phi d + q b + \beta EV(\tau, (1-\phi) d + b, y')$$

taking $b$ as given. If the government opts to default, the corresponding value function is:

$$V^1_2(h(\tau), d, y) = \max_{\tau} \{ u(\tau h(\tau) y) + \tau h(\tau) y - \phi d + q b + \beta EV(\tau, (1-\phi) d + b, y') \}$$

where

$$\tau \in \{0, 1\}$$

and $\tau$ is the value of $\tau$ that maximizes $EV(\tau, (1-\phi) d + b, y')$ for some $y'$.

This section extends the baseline model in order to incorporate this adverse outcome effect of fiscal contraction in a reduced form way. We assume the output is given by $y(h)$, where the function $h$ captures the negative effects of taxes on output loss. The function $h$ satisfies $h(0) = 1$, $h(1) = 0$, $h' < 0$ and $h'' < 0$. The assumption of $h'' < 0$ captures the idea of convex tax costs: larger tax rates lead to more distortions at the margins.

The basic logic underlying the time inconsistency problem in the baseline model remains intact because the optimal choice of tax rate internalizes its impact on output. The negative effect of tight fiscal policy on output leads to lower tax rates with and without commitment. As before, without commitment, the government remains unable to internalize the effects of $\tau$ on bond prices and hence the optimal tax under commitment $\bar{\tau}$ is greater than the time-consistent tax rate $\tau^*$.

In conclusion, the claim that fiscal policy in Europe should take into account the adverse effects of taxation on output does not invalidate the point that in the absence of an arrangement designed to overcome the time inconsistency problem, fiscal policy will all the same remain too loose. However, this paper has nothing to say on the comparison between optimal tax rates and those observed in the real world. Critics of the European austerity plans argue that fiscal policy has been excessively contractionary. Some economists even claim that current tax rates are at the wrong side of the Laffer curve and have actually reduced defaults more likely.28 Policy blunders may occur and official lenders (together with borrowers) may enact self-defeating fiscal packages, but this paper has nothing to add to this discussion.

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where $\tau$ maximizes

$$u(\tau h(\tau) y) + \tau h(\tau) y - \phi d + q b + \beta EV(\tau, (1-\phi) d + b, y')$$

taking $d' = (1 - \phi) d + b$ as given. $V^2$ is the maximum of two value functions:

$$V^2_1(h(\tau), d, y') = \max \{ V^1_1(h(\tau), d, y'), V^2_2(h(\tau), d, y') \}$$

Still in odd periods, if the government opts to default, the value function is:

$$V^2_1(h(\tau), d, y) = \max_{\tau} \{ u(\tau h(\tau) y) + \tau h(\tau) y - \phi d + q b + \beta EV(\tau, (1-\phi) d + b, y') \}$$

28 DeLong and Summers (2012) argue that with a modicum of hysteresis and reasonable values for the fiscal multiplier in the vicinity of the zero bound, fiscal contractions increase the debt burden.
where
\[
V_d(\tau, d, y) = u(1-\gamma)\cdot y' + \tau\cdot y' + \beta\left(1-\xi\right)EV^d(\tau', d, y') + \xi EV^d(\tau', 1d, y')
\]
and \(y' = (1-\gamma)\cdot y\) as before. As the default punishment kicks in one period after the default decision.

The value function associated with repaying debt in even periods is:
\[
V^e_d(\tau, d, y) = \max_b \left\{ u(1-\gamma)\cdot y + \tau y + q + \beta EV^d(\tau, (1-\gamma)d + b, y') \right\}
\]
where \(V^d\) is the maximum of two value functions:
\[
V^d(\tau', d, y') = \max\left\{ V^d(\tau', d', y'), V^d(\tau', d', y') \right\}
\]

If the government opts to default, the value function in even periods is:
\[
V^d(\tau, d, y) = u(1-\gamma)\cdot y + \tau y + \beta EV^d(\tau, d, y)
\]

where
\[
V^d(\tau', d, y') = \max\left\{ u(1-\gamma)\cdot y' + \tau' y' + \beta\left(1-\xi\right)EV^d(\tau', d, y') + \xi EV^d(\tau', 1d, y') \right\}
\]
and \(y' = (1-\gamma)\cdot y\) as before.

We want to understand the choice of \(\tau\), so we focus on the problem in odd periods. Consider the case that the debtor has access to capital markets. The optimal tax rate \(\tau\) in equilibrium comes from maximizing (3) taking the level of debt for the following period \(d'\) as given (owing to the assumptions on timing). The first order condition for \(\tau\) is simply:
\[
\frac{\partial EV^d(\tau', d, y')}{\partial \tau} = 0
\tag{14}
\]

which is similar to the expression in the basic model. However, it leads to a different result, because \(\tau\) will affect prices of debt in the following period.

Propositions 1, 2 and 3 still hold in this setting and the proofs are analogous. The following proposition shows that the tax rate under commitment \(\tilde{\tau}\) is also larger than the time-consistent tax rate \(\tau\) in this setting.

**Proposition 7.** In the model where the tax rate \(\tau\) can only be modified every other period, if a country under default risk can commit to a tax rate \(\tilde{\tau}\) when issuing debt, then \(\tilde{\tau} > \tau\).

**Proof.** See Appendix A.10.

As in the basic model, the first order condition when the government can commit to a tax rate \(\tilde{\tau}\) has an extra term that does not enter the condition in Eq. (14). This term captures the effect of taxes on the price of debt in the current period and leads to an extra incentive for tight fiscal policy. When choosing the tax rate \(\tau\), the government does not take into account the impact of this decision on the price of its current stock of debt — even though it internalizes the effect of \(\tau\) on the price of debt to be issued in the next period. Hence, qualitatively, the main result of the paper is unaffected. The next section shows that the result only vanishes in the limit of very short term debt.

### 7.3. Short term debt: a limiting result

We now assume that the government issues one-period debt but only changes \(\tau\) at the end of each period with probability \(\Psi\). With probability \(1-\Psi\), the tax rate prevailing in the following period \(\tau\) remains equal to \(\tau\). As before, the assumption of a low \(\Psi\) reflects either fiscal policy rigidities (the tax rate remains constant for a long time) or short term borrowing (debt is issued and repaid many times before fiscal policy decisions are made). As \(\psi \to 0\), period-by-period discretion on fiscal policy decisions converges into a predetermined tax rate.

The value functions now are:
\[
V(\tau, d, y) = \max_d \left\{ u(1-\gamma)\cdot y + \tau y - d + qd + \beta EV^d(\tau', d', y') \right\}
\]

where \(\psi = \tau\) with probability \(1-\Psi\) and, with probability \(\Psi\), the tax rate \(\tau\) maximizes
\[
u(1-\gamma)\cdot y + \tau y - d + qd + \beta EV^d(\tau', d', y')
\]

taking \(d'\) as given. If the government opts to default, the value function is:
\[
V_d(\tau, d, y) = \max_{\tau'} \left\{ u(1-\gamma)\cdot y + \tau y + \beta EV_0(\tau', d, y') \right\}
\]

with probability \(\Psi\) and
\[
V_d(\tau, d, y) = u(1-\gamma)\cdot y + \tau y + \beta EV_0(\tau, d, y')
\]

with probability \(1-\Psi\), where the expression for \(V_d(\tau, d, y)\) is analogously modified in order to fit the timing of tax changes of this section.

Propositions 1, 2 and 3 still hold in this setting and the proofs are analogous. The following proposition shows that the tax rate under commitment \(\tilde{\tau}\) is still larger than the time-consistent tax rate \(\tau\), but this difference vanishes in the limit as debt becomes shorter.

**Proposition 8.** In the model where the tax rate \(\tau\) can only be modified with probability \(\Psi\) in each period:

1. If a country under default risk can commit to a tax rate \(\tilde{\tau}\) when issuing debt, then \(\tilde{\tau} > \tau\).
2. As \(\psi \to 0\) and \(\beta \to 1\), the time consistent tax rate \(\tau\) converges to \(\tilde{\tau}\).

**Proof.** See Appendix A.10.

The case of \(\psi \to 0\) and \(\beta \to 1\) corresponds to a very short term debt (since the case of extreme fiscal policy rigidity would be captured by a very low \(\Psi\) but a value of \(\beta\) smaller than 1). Hence a very short term debt solves the time inconsistency problem. However, away from the limit, fiscal policy is still time inconsistent.

When choosing the tax rate \(\tau\), the government does not take into account the impact of this decision on the price of its current stock of debt, but it internalizes the effect of \(\tau\) on the price of debt to be issued \(t\) periods ahead with probability \((1-\psi)\). Hence short term debt attenuates the time inconsistency problem that leads to a low fiscal stance. As the probability of being able to reset taxes \(\Psi\) approaches zero, the government takes into account the effect of its tax decision on the price of debt in all future periods. As \(\beta\) approaches 1, the present period becomes unimportant, so \(\tau^* \to \tilde{\tau}\). But for any \(\psi > 0\) and \(\beta < 1\), the time-consistent tax rate \(\tau^*\) is lower than it would be under commitment.

Short-term debt in this model can thus be seen as a commitment device to tighter fiscal policy. However, short-term debt is also widely deemed to increase the risk of liquidity crises or sudden stops. Owing to the risks involved in short term borrowing, there is literature aiming to explain why emerging economies borrow short term. In Jeane (2009), for instance, the domestic government undertakes a fiscal adjustment only if threatened by a large withdrawal of external funds, and large and costly sudden reversals are only possible if debt is mostly short term. The model in this paper offers a different

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29 For empirical evidence on the negative effects of short term debt, see e.g. Sachs et al. (1996).
30 See, for example, Broner et al. (2013) and references therein.
31 See also Calvo and Guidotti (1992) and Niepelt (forthcoming).
explanation. Lower debt maturity means that a sizable amount of debt will be floated between two rounds of fiscal policy decisions, meaning that the effect of fiscal policy on the price of this debt is internalized by the government. Short term debt reduces the gains from trying to devalue debt. 

Nevertheless, owing to its drawbacks, short term debt might not be the best solution to the time inconsistency problem in fiscal policy. In that case, an international lender of last resort capable of providing cheap but senior lending as a reward for fiscal austerity might improve welfare.

8. Final remarks

There is a fast growing literature following in the footsteps of Eaton and Gersovitz (1981) that provides insights to some important questions about sovereign debt and default. However, most of this literature abstracts from fiscal policy and thus cannot address some of the main current policy issues. This paper studies incentives for fiscal adjustment in an otherwise standard model of sovereign default. Inefficently low fiscal adjustment arises as a result of imperfect commitment to repay debt, a usual assumption in the academic literature on sovereign risk. This suggests a novel interpretation for the role of lenders of last resort such as the IMF. Hence one contribution of this paper is to bring a well-established academic literature closer to policy discussions.

One implication of our model is that an excessively tight fiscal stance from an ex post point of view may be in the interest of the debtor ex ante. Another implication is that shocks that negatively affect the IMF’s lending capacity might push debtors away from a tight fiscal policy, which has the opposite flavor of the usual moral hazard critique associated with IMF lending.

The vast majority of dynamic models in the recent literature on sovereign debt and default are solved quantitatively. That is not surprising given the difficulties in the obtained closed form results in this literature. In our view, however, analytical results provide insightful results, even if that requires adding a few simplifying assumptions to the models. Ultimately, we see both approaches as complements and quantitative versions of this model could provide more insight on the incentives for fiscal adjustment for a debtor government facing the risk of default.

Appendix A

A.1. First-best with contingent taxation

Now we characterize the first-best allocation in which the debtor country has access to contingent taxes—that is, it can set the tax rate upon observing the realization of output and can issue contingent debt. With this structure a costly default never occurs because repayment can be agreed to be zero in some states of nature. Given that \( \beta (1 + r') < 1 \) and the utility is linear in the consumption of the public good, a benevolent government should set \( g_i = 0 \) for all \( i > 1 \). Taking this into account, it solves the following problem:

\[
\begin{align*}
    u(c_1) + g_1 + \sum_{t=2}^{\infty} \beta^{t-1} u(c_t)
\end{align*}
\]

subject to:

\[
\begin{align*}
    \sum_{t=1}^{\infty} \frac{c_t}{(1 + r')^{t-1}} + g_1 &= \sum_{t=1}^{\infty} \frac{y_t}{(1 + r')^{t-1}}.
\end{align*}
\]

Given the assumption \( u'(y(t)) < 1 \) and the Inada condition, \( u'(c_t) = 1 \). The intertemporal FOC in turn is:

\[
\begin{align*}
    1 = (\beta (1 + r'))^{t-1} E(u'(c_{t+1})) \quad \text{for all } t \geq 2.
\end{align*}
\]

Since \( \beta (1 + r') < 1 \), \( u'(c_{t+1}) > u'(c_t) \) for all \( t \), meaning private consumption falls over time (but concavity prevents it from being zero for all \( t \geq 2 \)).

A.2. Lemma 9

Lemma 9. For given \( \tau \) and \( y \), there exists \( \bar{d} \geq 0 \) such that there is a default in the current period if and only if \( d > \bar{d} \).

Proof. The value function in case of default is not affected by \( d \). In case of repayment, an increase in \( d \) reduces \( g \) by the same amount if the government budget constraint is not binding and is at least as bad if the constraint is binding. Hence:

\[
\frac{\partial V_p}{\partial d} = 0 \quad \text{and} \quad \frac{\partial V_p}{\partial d} \leq -1.
\]

If \( d' = 0 \), \( V_p > V_p(0) \) for all \( y \) and \( \tau \) because all possible \( g \) under default are also possible under repayment (repayment includes the set of options available in case of default) and defaulting leads to a direct utility loss. For high enough \( d \), consumption conditional on repayment approaches 0, hence the marginal utility of consumption and, consequently, \( V_p \) can be arbitrarily low, so \( V_p < V_d \). Combining all those statements yields the claim.

A.3. Proof of proposition 1

Taking the derivatives of the value functions, and applying the envelope theorem with respect to \( b \) and \( \tau \) yields the derivatives. That shows that the difference between value functions \( V_p \) and \( V_d \) is not affected by \( y \). Since there are no other shocks in the model, agents know in advance whether debt will be honored or not. Since the price of debt falls to 0 if debt will not be repaid, and it is never optimal for a country to sell bonds at zero price, the default never occurs.

A.4. Proof of proposition 2

A.5. Proof of the first statement

First, we show that when the constraint \( g \geq 0 \) is locally binding,

\[
\frac{dV_p(\tau, d, y)}{dy} > u'(y(1-\tau))(1-\tau) + \tau.
\]

Since the constraint on \( g \) is binding,

\[
V_p(\tau, d, y) = \max_{b, \tau'} u(y(1-\tau')) + \beta E[V(\tau', (1-\tau)d + b, y')], \quad \text{with } q_b = \phi d - \tau y
\]

32 A related mechanism is at work in Misarle and Blanchard (1994). As debt increases so does the temptation to devalue it via surprise inflation. To maintain its anti-inflationary credibility, the government then shortens the maturity of its obligations thus rendering the opportunistic strategy unfeasible.

33 Fink and Scholl (2014) provide a recent example of quantitative work along these lines. They study how an International Financial Institution that provides subsidized loans and imposes fiscal conditionalities affects government decisions on sovereign debt and default.
thus
\[
\frac{\partial V_p(\tau, d, y)}{\partial y} = u'(y)(1-\tau)(1-\tau) + \beta \frac{\partial E[V(\tau', (1-\psi)d + b, y')]}{\partial b} \bigg|_{b=(\psi-d-\tau)/q}.
\] (16)

We need show that the second term is larger than \(\tau\). Note that
\[
\frac{\partial (qb)}{\partial y} \bigg|_{b=(\psi-d-\tau)/q} = \frac{\partial (qb) \partial b}{\partial b} \bigg|_{b=(\psi-d-\tau)/q}
\]
and since
\[
\frac{\partial (qb)}{\partial b} \bigg|_{b=(\psi-d-\tau)/q} = -\tau \quad \text{and} \quad \frac{\partial (qb)}{\partial b} = q(1-\epsilon)
\]
we have
\[
\frac{\partial b}{\partial y} \bigg|_{b=(\psi-d-\tau)/q} = -\frac{\tau}{q(1-\epsilon)}.
\] (17)

Moreover, since the constraint is locally binding, \(b = (\psi-d-\tau)/q\) and the derivative of the value function with respect to \(b\) is negative (the country would prefer a smaller \(b\) leading to a smaller \(g\) but has to borrow enough to get \(g = 0\)). Therefore
\[
q(1-\epsilon) < -\beta E[V(\tau', (1-\psi)d + b, y')].
\] (18)

Combining Eqs. (17) and (18) yields:
\[
\beta E[V(\tau', (1-\psi)d + b, y')] \bigg|_{b=(\psi-d-\tau)/q} > \tau.
\]

Using Eq. (16) implies the condition in Eq. (15). When \(g > 0\), \(dV_p(\tau, d, y)/dy\) as is given by Eq. (1). Moreover, \(dV_g(\tau, d, y)/dy\) is always given by Eq. (1). Hence
\[
\frac{dV_p(\tau, d, y)}{dy} > \frac{dV_g(\tau, d, y)}{dy}.
\] (19)

There cannot be a default for all \(y\) since that implies \(q = 0\) in the previous period, which is never possible.

Suppose that there is a default for some \(y_1 \leq (y, y_1)\), which implies \(V_d(\tau, d, y_1) > V_p(\tau, d, y_1)\). Since there cannot be a default for all \(y\), there exists \(y_2\) such that \(V_d(\tau, d, y_2) \leq V_p(\tau, d, y_2)\). Define \(y\) as the smallest value of \(y\) such that \(V_d(\tau, d, y) \leq V_p(\tau, d, y)\) which exists because the value functions are continuous in \(y\). In that case there is a default for all \(y < \hat{y}\).

Lastly, no default for \(y = \hat{y}\) implies no default for all \(y > \hat{y}\), because owing to Eq. (19), \(V_p(\tau, d, y) \geq V_d(\tau, d, \hat{y})\) implies \(V_p(\tau, d, y) \geq V_d(\tau, d, y)\) for all \(y > \hat{y}\).

A.6. Proof of the second statement

When the constraint \(g \geq 0\) is slack,
\[
\frac{dV_p(\tau, d, y)}{d\tau} = \frac{dV_g(\tau, d, y)}{d\tau} = u'(y)(1-\tau)(1-\tau) + y.
\] (20)

If the constraint on \(g\) is binding,
\[
\frac{dV_p}{d\tau} = u'(y)(1-\tau)(1-\tau) + \beta \frac{\partial E[V(\tau', (1-\psi)d + b, y')]}{\partial b} \bigg|_{b=(\psi-d-\tau)/q}.
\]

First we show that
\[
\beta \frac{\partial E[V(\tau', (1-\psi)d + b, y')]}{\partial b} \bigg|_{b=(\psi-d-\tau)/q} > \tau.
\]

Repeating the steps of the proof of Proposition 2 that lead to Eq. (17), we have
\[
\frac{\partial b}{\partial \tau} \bigg|_{b=(\psi-d-\tau)/q} = -\frac{\tau}{q(1-\epsilon)}
\]
and using Eq. (18) we obtain the expression in Eq. (20).

An argument similar to that in proposition 2 establishes the second statement.

A.6.1. Proof of the third statement

If the probability of a default next period is strictly positive, at the smallest value \(y\) such that the debtor is indifferent between defaulting or not (call it \(\hat{y}(\tau, d) \equiv (y_0, y_1)\)) the value function \(V_p\) crosses \(V_g\) so
\[
\frac{dV_p(\tau, d, y)}{dy} \bigg|_{y=\hat{y}(\tau, d)} > \frac{dV_g(\tau, d, y)}{dy} \bigg|_{y=\hat{y}(\tau, d)} = u'(y)(1-\tau)(1-\tau) + \tau
\]
which can only be true if Eq. (18) holds and implies that the constraint on \(g\) is binding. But that implies that the expression in Eq. (20) which leads to
\[
\frac{dV_p(\tau, d, y)}{d\tau} > \frac{dV_g(\tau, d, y)}{d\tau} \bigg|_{y=\hat{y}(\tau, d)}
\]
so an increase in \(\tau\) makes \(V_p(\tau, d, y(\tau, d))\) strictly larger than \(V_g(\tau, d, y(\tau, d))\). Using the results in the first statement of this proposition, the smallest value \(y\) such that the debtor is now indifferent between repaying and defaulting is smaller than \(\hat{y}(\tau, d)\). Since the distribution of \(y\) has full support in \([y_0, y_1]\) the probability of the default next period becomes smaller when \(\tau\) increases.

A.7. Proof of Proposition 3

If the price of debt is conditional on \(\tau\), then using Eq. (1) we get
\[
\frac{\partial q}{\partial \tau} = \frac{\partial q_1}{\partial \tau} \left( 1 + \tau \right) + \frac{1}{(1 + \tau)} \left( \zeta \phi(1-\psi)q(d') \right) + \frac{1}{(1 + \tau)^2} \left( \phi(1-\psi)q(d') \right)
\]

The third statement of Proposition 2 establishes that the probability that debt will be repaid next period is increasing in \(\tau\) \((\partial q_1/\partial \tau > 0)\) as long as there is a positive probability of default and the term in brackets is positive owing to the assumption \(\phi > \nu\), since \(q(d') < 1\) and \(\zeta (\psi + \tau) < 1\). Hence the first term in the expression for \(\partial q/\partial \tau\) is positive.

A higher \(\tau\) increases the expected revenues for the following period, so it might relax the budget constraint. In case the debtor chooses to default in the following period, \(q(d')\) is not affected. If the debtor does not default, higher revenues will mean either (i) a lower \(d'\) in case the constraint \(g \geq 0\) is binding so that a larger \(\tau\) allows for a lower \(b\); or (ii) the same \(d'\) in case the constraint \(g \geq 0\) was not binding. In either case \(\partial q(d')/\partial \tau \geq 0\).

A.8. Proof of proposition 4

We want to compare the choices at \(t_0\) of a government (i) in case it can commit to a tax rate \(\tau\) when issuing debt in period \(t_0\); and (ii) in case it chooses a tax rate \(\tau\) at \(t_0\) after debt has been contracted. The comparison takes as given the possibility of commitment in the future, so
consider that the country can commit to choose a certain tax rate \( \tau_{t+1} \) when issuing debt \( b_t \) in periods \( t \in T_C \), where \( T_C \subset \{ t_0 + 1, t_0 + 2, \ldots, \infty \} \), and cannot commit otherwise.

The value function for a government that chooses to repay debt at \( t \geq t_0 \) in case the government can commit (for \( t \in T_C \)) and in case of commitment at \( t_0 \) is:

\[
V_t^G (\tau, d_t, y_t) = \max_{t \in T_C} \{ u((1-\tau_t) y_t) + \tau_t y_t - \psi d_t + q_t b_t + \beta E V_{t+1}(\tau_{t+1}, d_{t+1}, y_{t+1}) \}
\]

where \( d_{t+1} = (1 - \varphi) d_t + b_t \). In this case, the price of debt \( q_t \) is a function of \( \tau_{t+1} \) (and \( d_{t+1} \) as well). The expression for \( q_t \) is still given by an expression like Eq. (1) but now changes in \( \tau \) affect the price of debt through the probability of default and the price of debt in the following period (\( \tau_1 \) and \( q(d') \)).

In case commitment is not possible (for \( t \in T_C \)) and in case of no commitment at \( t_0 \), the value function for a government that chooses to repay debt is:

\[
V_t^G (\tau, d_t, y_t) = \max_{t \in T_C} \{ u((1-\tau_t) y_t) + \tau_t y_t - \psi d_t + q_t b_t + \beta E V_{t+1}(\tau_{t+1}, d_{t+1}, y_{t+1}) \}
\]

where \( d_{t+1} = (1 - \varphi) d_t + b_t \) and \( \tau_{t+1} \) maximizes

\[
u_t(1 - \tau_t) y_t + \tau_t y_t - \psi d_t + q_t b_t + \beta E V_{t+1}(\tau_{t+1}, d_{t+1}, y_{t+1})
\]

taking \( d_{t+1} \) and the price of debt \( q_t \) as given.

The value function in the case of default in period \( t \) does not depend on whether \( t \in T_C \) (since no debt is issued in period \( t \)). Hence the value function of default is given by

\[
V_t^D (\tau, d_t, y_t) = \max_{t \in T_C} \{ u((1-\tau_t) y_t) + \tau_t y_t + \beta E V_{t+1}(\tau_{t+1}, d_{t+1}, y_{t+1}) \}
\]

where

\[
V_t^D (\tau, d_t, y_t) = \max_{t \in T_C} \{ u((1-\tau_t) y_t) + \tau_t y_t + \beta E V_{t+1}(\tau_{t+1}, d_{t+1}, y_{t+1}) \}
\]

and \( y_{t+1} = (1 - \gamma) y_t + y_t \). The default punishment kicks in one period after the default decision.

Finally, the value function \( V_{t+1} \) is given by

\[
V_{t+1}(\tau, d_{t+1}, y_{t+1}) = \max \{ V_t^G(\tau, d_t, y_t), V_t^D(\tau, d_t, y_t) \}
\]

in case \( t + 1 \in T_C \) and is

\[
V_{t+1}(\tau, d_{t+1}, y_{t+1}) = \max \{ V_t^G(\tau, d_t, y_t), V_t^D(\tau, d_t, y_t) \}
\]

in case \( t + 1 \notin T_C \).

Value functions are functions of \( t \) because the possibility of commitment in the future may vary in time. However, the expected value function \( E V_t^G \) is the same in the expressions for \( V_t^G \) and \( V_t^D \).

Propositions 1, 2 and 3 hold in this case and except for a change in notation, the proofs are the same. We now prove Proposition 4.

Define \( V_t^G \) as the maximand in \( V_t^G(\tau, d_t, y_t) \):

\[
\frac{\partial V_t^G(\tau, d_t, y_t)}{\partial \tau} = 0 \text{ and } \frac{\partial V_t^G(\tau, d_t, y_t)}{\partial \tau} = 0.
\]

Without commitment,

\[
\frac{\partial V_t^N(\tau, d_t, y_t)}{\partial \tau} = \beta E V_{t+1}(\tau, d_{t+1}, y_{t+1})
\]

With commitment, repeating the steps in Section 4, we obtain

\[
\frac{\partial V_t^N(\tau, d_t, y_t)}{\partial \tau} = \beta E V_{t+1}(\tau, d_{t+1}, y_{t+1})
\]

in case the constraint \( g \geq 0 \) does not bind in period \( t_0 \) and

\[
\frac{\partial V_t^N(\tau, d_t, y_t)}{\partial \tau} = \beta E V_{t+1}(\tau, d_{t+1}, y_{t+1})
\]

case the constraint \( g \leq 0 \) binds. Importantly, the function \( EV_{t+1} \) is the same regardless of the possibility of commitment in the current period. The difference in case of commitment is given by an extra positive term in the derivative related to the effect of \( \tau \) on the price of debt.

The remaining of the proof is done by contradiction. First consider the unconstrained case, given by Eq. (22). Suppose that \( \tau = \tau^* \). Then \( b_t \) and, consequently, \( d_{t+1} \) are the same in both cases (with and without commitment) since \( b_t \) is given by the same equation. Using Proposition 3 and as \( b_t > 0 \) by assumption, \( \frac{\partial V_t^D}{\partial \tau} > 0 \) implies that \( \frac{\partial V_t^D}{\partial \tau} > 0 \), which is a contradiction. If \( \tau \) is smaller than in the case without commitment, then the second term in Eq. (22) is positive and as the first term is also positive, \( \frac{\partial V_t^D}{\partial \tau} > 0 \), which is a contradiction. Now consider the constrained case given by Eq. (23). The first term of the derivative is positive (the derivative inside the integral is negative and all other terms are positive with a negative sign). Again, if \( \tau \) is chosen to be equal or smaller than the case without commitment, \( \frac{\partial V_t^D}{\partial \tau} > 0 \) at the optimal \( \tau^* \) is a contradiction.

Note that different assumptions on the commitment set \( T_C \) would only affect the expressions in Eqs. (21), (22) and (23) through their effects on the function \( EV_{t+1} \). Since Proposition 2 holds for any set \( T_C \), different assumptions on commitment do not affect the proof of the time-consistency problem. For the sake of notational simplicity, Propositions 5, 6 and 7 consider the case \( T_C = \emptyset \).

A.9. Proof of Proposition 5

A.9.1. First statement

After debt has been contracted, the optimal tax rate is \( \tau^* \). A simple application of the envelope theorem tells us that choosing \( \tau^* + \Delta \tau \) for a small \( \Delta \tau \) yields only second-order losses. Thus a transfer \( T \) of a second order of magnitude is enough to implement \( \tau^* + \Delta \tau \). The contract entails a loan from debtor to creditor at the risk-free rate, but this amount (and thus the increase in the amount borrowed by the debtor to cover this loan) is also of second order of magnitude.

Now a corollary of Proposition 4 is that a country can commit to a tax rate \( \tau \), then:

\[
\frac{\partial V_t^G(\tau, d_t, y_t)}{\partial \tau} > 0
\]
where this derivative is given by Eq. (11) or Eq. (12), depending on whether the constraint \( g \geq 0 \) is binding or not. Hence an increase in \( \tau \) leads to a first order increase in the value function of the debtor country. This increase takes into account that lenders break even, which yields the first claim.

A.9.2. Second statement
For risk neutral lenders, paying a constant transfer or a contingent transfer makes no difference as long as the expected value of transfers is the same. Hence we need to show that the positive effect of a transfer in a state where \( g = 0 \) on welfare in the debtor country is larger than the positive effect of a transfer with the same magnitude in a state where \( g > 0 \).

If the debtor is not constrained, \( g > 0 \), then

\[
\frac{\partial V_p(\tau, d, y)}{\partial \tau} = 1.
\]

Now suppose the debtor is constrained and receives a transfer \( T \), so that \( \tau y + T + qb - \varphi d = 0 \). Then

\[
b = \frac{\varphi d - \tau y - T}{q}.
\]

Now

\[
\frac{\partial (qb)}{\partial \tau} \bigg|_{\tau = (\varphi d - \tau y - T)/q} = \frac{\partial (qb)}{\partial \tau} \bigg|_{\tau = (\varphi d - \tau y - T)/q}
\]

and since

\[
\frac{\partial (qb)}{\partial \tau} \bigg|_{\tau = (\varphi d - \tau y - T)/q} = -1 \quad \text{and} \quad \frac{\partial (qb)}{\partial \tau} = q(1 - \epsilon)
\]

we get

\[
\frac{\partial b}{\partial \tau} \bigg|_{\tau = (\varphi d - \tau y - T)/q} = \frac{1}{q(1 - \epsilon)}.
\]  
(24)

Moreover

\[
\frac{\partial V_p(\tau, d, y)}{\partial \tau} = \beta \frac{\Delta E[V(\tau', d', y')] \partial b}{\partial \tau} \bigg|_{\tau = (\varphi d - \tau y - T)/q}.
\]  
(25)

If the constraint is binding, Eq. (18) holds. Combining it with Eqs. (24) and (25) yields:

\[
\frac{\partial V_p(\tau, d, y)}{\partial \tau} > 1.
\]

A.10. Proof of Proposition 6

A.10.1. Proof of the first statement
Now, as long as the constraint \( g \geq 0 \) is slack,

\[
\frac{dV_p(\tau, d, y)}{dy} = \frac{dV_d(\tau, d, y)}{dy} = u'(h(\tau)y(1 - \tau))(1 - \tau)h(\tau) + \tau h(\tau).
\]

When the constraint \( g \geq 0 \) is locally binding, \( b = (\varphi d - \tau h(\tau)y)/q \). Repeating the same steps as in the proof of Proposition 2, we get

\[
\frac{\partial b}{\partial \tau} \bigg|_{\tau = (\varphi d - \tau h(\tau)y)/q} = \frac{-\tau h(\tau)}{q(1 - \epsilon)}
\]

and since Eq. (18) also holds in this case,

\[
\frac{dV_p(\tau, d, y)}{dy} = u'(h(\tau)y(1 - \tau))(1 - \tau)h(\tau) + \tau h(\tau)
\]

and the argument in the proof of Proposition 2 yields the claim.

A.10.2. Proof of the second statement
When the constraint \( g \geq 0 \) is slack,

\[
\frac{dV_p(\tau, d, y)}{\partial \tau} = \frac{dV_d(\tau, d, y)}{\partial \tau} = u'(h(\tau)y(1 - \tau))\left(\frac{\partial (h(\tau)\tau)}{\partial \tau} + y \frac{\partial (h(\tau)\tau)}{\partial \tau}\right)
\]

If the constraint on \( g \) is binding,

\[
\frac{dV_p(\tau, d, y)}{\partial \tau} = u'(h(\tau)y(1 - \tau))\left(\frac{\partial (h(\tau)\tau)}{\partial \tau} + y \frac{\partial (h(\tau)\tau)}{\partial \tau}\right) + \beta \frac{\partial E[V(\tau', (1 - \varphi)d + b, y')] \partial b}{\partial \tau} \bigg|_{\tau = (\varphi d - \tau y - T)/q}.
\]

Repeating the steps of the proof of Proposition 2 that lead to Eq. (17), we have

\[
\frac{\partial b}{\partial \tau} \bigg|_{\tau = (\varphi d - \tau y - T)/q} = -\frac{y}{q(1 - \epsilon)} \frac{\partial (h(\tau)\tau)}{\partial \tau}
\]

and here, the fact that \( \frac{\partial (h(\tau)\tau)}{\partial \tau} > 0 \) plays a key role. The above equation and the expression in Eq. (18) imply:

\[
\beta \beta \frac{\partial E[V(\tau', (1 - \varphi)d + b, y')] \partial b}{\partial \tau} \bigg|_{\tau = (\varphi d - \tau y - T)/q} > \frac{\partial (h(\tau)\tau)}{\partial \tau}.
\]

The proofs of the third, fourth and fifth statements are analogous to the proof of the third statement of Proposition 2 and the proof of Proposition 3, noting that \( \tau h(\tau) \) is increasing in \( \tau \) in the interval \([0, \tau]\).

A.11. Proof of Proposition 7

Suppose the government can commit to some tax rate \( \tau \) when it issues debt \( b \). The value function in case of default is unchanged as in Eq. (4), but the value function in case of repayment becomes

\[
V_p(\tau, d, y) = \max_b \left\{ u(\varphi d - \tau y - \varphi d + q) + \beta E[V(\tau', (1 - \varphi)d + b, y')] \right\}
\]  
(26)

and \( q \) is now a function of both \( d' \) and \( \tau \). As before, and not surprisingly, \( \tau \) positively affects the price of debt:

\[
\frac{\partial q}{\partial \tau} > 0.
\]

We now show that if the debtor could commit to a certain tax rate \( \tilde{\tau} \), the chosen \( \tilde{\tau} \) would be larger than \( \tau \) given by Eq. (9). In this case, \( \tilde{\tau} \) maximizes the expression for \( V_p(\tau, d, y) \) in Eq. (26), taking into account the effect of \( \tau \) on \( q \). The solution to the maximization problem for \( \tilde{\tau} \) depends on whether the constraint \( g \geq 0 \) binds. Suppose first it does not bind. Then \( \tau y - \varphi d + q b > 0 \) and after some algebra, using the first order condition with respect to \( b \), we get at:

\[
\frac{\partial V_p(\tau, d, y)}{\partial \tau} = \frac{\partial q}{\partial \tau} + \beta \frac{\partial E[V(\tau', d', y')] \partial d}{\partial \tau}.
\]

The difference between this and Eq. (9) is the first term, which is positive, which leads to \( \tau > \tilde{\tau} \).
Now suppose the constraint \( g \geq 0 \) binds so that \( \tau y - \varphi d + q b = 0 \). So the derivative of \( b \) with respect to \( \tau \) is

\[
\frac{\partial b}{\partial \tau} = -\frac{(\varphi d - \tau y) \partial q}{q^2 \frac{\partial \tau}{\partial \tau}}
\]

which implies

\[
\frac{\partial V_1^t(\tau, d, y)}{\partial \tau} = \beta \left( \frac{(\varphi d - \tau y) \partial q}{q^2} \int_q \frac{\partial V_1^t(\tau, (1-\varphi)d + b, y')}{\partial b} f(y')dy' + \frac{\partial EV_2(\tau, d, y')}{\partial \tau} \right)
\]

Again, the difference between this and Eq. (9) is the first term, which is positive because \( \partial V_1^t/\partial b < 0 \). Thus \( \tau > \tau^* \).

A.12. Proof of Proposition 8

The derivative of the value function \( V_1^t \) with respect to the time-consistent tax rate \( \tau^* \) at time \( t \) is given by:

\[
j = \beta \frac{\partial EV(\tau^*, d_{e+1}, Y_{e+1})}{\partial \tau} + \sum_{s=1}^n \beta^{s-1}(1-\psi)^{-1} \left( \frac{\partial E(q_{s+1}d_{e+1})}{\partial \tau} + \beta \frac{\partial EV(\tau^*, d_{e+1}, Y_{e+1})}{\partial \tau} \right)
\]

while the derivative of the value function \( V_1^t \) with respect to the tax rate \( \tau \) under commitment is given by:

\[
j = \sum_{s=1}^n \beta^{s-1}(1-\psi)^{-1} \left( \frac{\partial E(q_{s+1}d_{e+1})}{\partial \tau} + \beta \frac{\partial EV(\tau, d_{e+1}, Y_{e+1})}{\partial \tau} \right)
\]

The sum of the weights on each period \( \beta^s (1-\psi) = \frac{1}{1-\beta(1-\psi)} \) is given by

\[
\frac{1}{1-\beta(1-\psi)} \frac{\partial q_{i+1} d_{i+1}}{\partial \tau}
\]

as \( \beta \to 1 \) and \( \psi \to 0 \), the difference \( j - j^* \) approaches 0 (since \( 1 - \beta(1-\psi) > 0 \)), hence \( \tau^* \to \tau \). This proves the second statement. The proof of the first statement is analogous to the argument leading to

Proposition 4, with these expressions for the derivative of the value function \( V_1^t \) with respect to \( \tau^* \) and \( \tau \).

References

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